Design for Global Damage Tolerance and Associated Mass Penalties

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A structural design with global damage tolerance is defined as a design that can tolerate the destruction of one or more major structural components. The mass penalty associated with improving the global damage tolerance of optimized structures is evaluated herein for structures typically used in aircraft wing construction. It is shown that this mass penalty is strongly related to the degree of redundancy of the structure, being most severe for structures of low redundancy. For highly redundant wing-box structures made of composite materials, it is shown that significant improvement in global damage tolerance may be achieved without any mass penalty.

Introduction

AIRCRAFT structures are designed to satisfy loading conditions based on normal and specified abnormal operating conditions. In addition, some designs must be able to sustain primary structural component damage to satisfy certification requirements; and other designs must be able to sustain a certain amount of battle damage and retain limited operational capability. Designing primary structural components to tolerate reasonable amounts of damage is not an exact science because some of the factors that can affect structural performance (e.g., foreign object damage and fatigue) are uncertain. Nevertheless, damage-tolerant design philosophies based on experience have evolved over the years for metal aircraft. A good review of recent work on assessing and improving the damage tolerance of aircraft structures is included in Ref. 1.

Although advanced-composite materials offer an attractive potential for reducing aircraft structural mass, composite structural components can be more sensitive to damage than metal components. For example, tests have shown that heavily loaded graphite-epoxy compression panels,² typical of stiffened wing-skin construction, can be sensitive to low-speed impact damage. Test specimens designed to satisfy conventional strength and buckling constraints failed prematurely at applied strains well below their design strains when subjected to impact damage. Sensitivity to damage becomes an even greater concern for optimized composite structures because optimized structures in general are often sensitive to damage.³

Because of this sensitivity to damage it is important to design composite aircraft structures with damage tolerance as one of the design constraints. Designing for damage tolerance can be done on the local level by requiring resistance to crack or damage propagation. It is also possible to provide global damage tolerance in a structure by requiring that the structure carry a percentage of the design load when a major structural component is destroyed.

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The present paper is concerned with optimum design for global damage tolerance with emphasis on the mass increment required to achieve such damage tolerance. Two simple structural configurations are used to illustrate the basic aspects of designing for damage tolerance. A wing-box example is then used to focus on design for damage tolerance of a highly redundant structure. For all three examples, reference optimum designs that have to carry prescribed loads without considering damage are obtained. Then, comparable optimum damage-tolerant designs are obtained. In the undamaged state, the damage-tolerant designs have to carry the same loads as the reference optimum designs with no damage. In the damaged state, these designs must carry a given percentage of the prescribed loads. The examples show how the mass of the damage-tolerant designs depend on the redundancy of the structure and on the percentage of load required to be carried in the damaged configuration.

Simple Design Examples

Delamination of a Plate in Compression

It has been shown4 that buckling of local delaminated regions caused by low-speed impact damage is a major contributor to premature failure of compression loaded graphite-epoxy panels. This delamination phenomenon is the motivation for a simple design example of a laminated plate subject to a uniaxial compression load N (see Fig. 1) that illustrates some of the aspects of designing for damage tolerance. The plate is assumed to be composed of a large number of layers which makes the laminate isotropic. The mode of damage is delamination where a plate of thickness tseparates into two thinner plates of thicknesses t_1 and t_2 . The design problem is simplified here by considering buckling to be the only design constraint. For the buckling load calculations, it is assumed that the plate could be considered infinitely wide and clamped at the loaded ends. The buckling load is then

$$N = \pi^2 E t^3 / 3 (1 - \nu^2) L^2 \tag{1}$$

where E is the effective longitudinal laminate modulus and ν is the laminate Poisson's ratio.

The required thickness for carrying a load N with no damage-tolerance considerations is

$$t = [3(1-\nu^2)NL^2/\pi^2E]^{1/3}$$
 (2)

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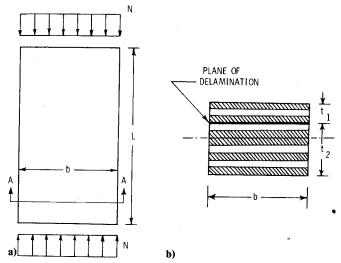


Fig. 1 Geometry and loading of rectangular plate delamination example: a) geometry and loading; b) view A-A showing laminate cross section.

This design, called herein the reference design, has a measure of damage tolerance. That is, when damaged it can carry a fraction, f, of the original undamaged buckling load (the buckling load in the damaged condition is fN). To calculate f, assume that owing to delamination the plate divides into two plates with thicknesses t_1 and t_2 (where $t_1 \le t_2$) and let

$$t_1/t_2 = \beta \qquad 0 \le \beta \le I \tag{3}$$

The residual strength, that is, the maximum load that this damaged structure can carry, is calculated based on two simplifying assumptions.

First, because of the large number of plies the effective modulus and Poisson's ratio for the two plates are assumed to be the same as for the original plate. Second, the analysis is simplified by assuming zero postbuckling stiffness. That is, after the thinner plate buckles it continues to carry the buckling load, but no more. Under these assumptions, the total buckling load is the sum of the individual buckling loads of the two plates and the residual strength is fN, where

$$r = (I + \beta^3) / (I + \beta)^3$$
 (4)

The worst condition corresponds to the minimum level of residual strength, which occurs when $\beta = 1.0$ and f = 0.25.

If a higher level of residual strength is desired, a different design (i.e., a thicker plate) must be used. It is assumed that delamination still occurs at $t_1/t_2 = \beta$ for a thicker plate. Then to provide for the higher level of residual strength the thickness of the plate must be increased by a factor μ . For values of f larger than the one obtained from Eq. (4), μ is given as

$$\mu = [f(I+\beta)^3/I + \beta^3]^{1/3}$$
 (5)

Values of μ are plotted as a function of f and β in Fig. 2, which shows the influence of the level of damage (represented by β) and the required level of residual strength (represented by f) on the mass. For example, for f=1.0 and $\beta=1.0$, $\mu=1.59$ so that the mass penalty for damage tolerance is 59%.

This simple example displays some important aspects of designing for damage tolerance. The first aspect is the uncertainty with respect to the location of damage, which is represented here by the parameter β . One common way of coping with such uncertainty is to design for worst-case damage (i.e., $\beta = 1$). The second aspect is that in a redundant structure there is some inherent measure of damage tolerance. That is, the damaged structure has some level of residual

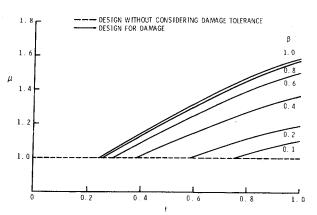
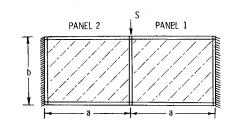


Fig. 2 Variation of normalized structural mass μ with required residual load factor f for different values of $\beta = t_1/t_2$.



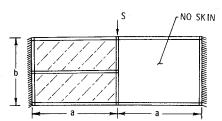


Fig. 3 Two-bay deep beam: a) original concept; b) alternate concept.

strength. If that level is satisfactory, no changes in the design are needed. If increased levels of residual strength are required, the design has to be changed, usually with increased mass.

This example, does not involve an optimized structure and, as a result, does not show the effect of optimization on the damage tolerance and the mass of the design. This effect is shown by the next example.

Two-Bay Built-Up Deep Beam

The second example shows the effect of the interaction of several components on the damage tolerance of the overall structure. A simple two-bay built-up deep-beam configuration (see Fig. 3a) loaded in shear is chosen to illustrate this interaction. In this example, it is assumed that the mode of damage is the destruction of either one of the shear panels. For this example, the panels are designed to prevent shear buckling. The external frame and interior stiffener are assumed to be fixed in size so that only the thicknesses of the shear panels may be varied. The total shear load that can be carried by a single panel is

$$S = \kappa \pi^2 E t^3 / 12b (1 - \nu^2) \tag{6}$$

where κ is a shear buckling coefficient that depends on the boundary conditions and the aspect ratio of the panel; E and ν are Young's modulus and Poisson's ratio; and b and t are the depth and thickness of the panel, respectively.

For the purpose of this illustration, the analysis is greatly simplified by assuming zero postbuckling stiffness. That is, after buckling the panel continues to carry the buckling load, but no more. This assumption is very conservative for panel buckling. It is also assumed that the configuration collapses after both panels have buckled. The collapse load of the structure is therefore taken to be the sum of the buckling loads of the two panels, and is given in terms of the thicknesses of the two panels t_1 and t_2

$$S_c = \kappa \pi^2 E(t_1^3 + t_2^3) / 12b(1 - \nu^2)$$
 (7)

The total mass of the two panels is

$$m = \rho ba(t_1 + t_2) \tag{8}$$

where ρ is the mass density and a is the length of each of the two panels.

To find the mass penalty associated with a given level of damage tolerance, the reference design (i.e., the optimum design with no damage-tolerance constraints) is found first. The minimum-mass design for no damage tolerance is found to correspond to one panel having zero thickness (say, panel 1 in Fig. 3a) and the other having a thickness of

$$t_{\theta} = [12Sb(1 - v^2) / \kappa \pi^2 E]^{1/3}$$
 (9)

Note that the optimized design is not redundant and has, therefore, no damage tolerance. The corresponding mass is

$$m_0 = \rho a b t_0 \tag{10}$$

Next, the design of the structure for damage tolerance is considered. The required damage tolerance is defined in terms of a residual strength fraction f. The structure is required to carry the design load S in its undamaged condition and a load fS in the damaged condition. The damaged condition is defined as the failure of either one of the two panels. For f=0, the minimum mass design is the reference design which has only one panel, $t_1=0$. For nonzero values of f, the optimum design consists of both panels. However, for small values of f, panel 2 is still much thicker than panel 1. The thickness of panel 1 is determined from the condition that it must carry the load fS when panel 2 fails owing to damage (worst-case damage). Then from Eq. (6),

$$t_{I} = [12fSb(I - \nu^{2}) / \pi^{2} \kappa E]^{1/3}$$
 (11)

The thickness of panel 2 is determined by the requirements that it carry (1-f)S when panel 1 is undamaged and fS when panel 1 is damaged so that

$$t_2 = [12f_m Sb(I - \nu^2) / \pi^2 \kappa E]^{1/3}$$
 (12)

where

$$f_m = \max[f, (1-f)]$$

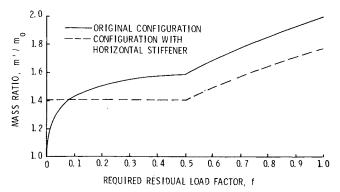


Fig. 4 Ratio of mass of damage-tolerant designs to reference designs for deep-beam example.

The ratio m/m_0 of the mass of the damage-tolerant design to the mass of the reference design is shown by the solid curve in Fig. 4, which consists of three distinct regions. The first region, from f=0 to about 0.2, is very steep. Thus obtaining the first 20% of residual strength requires a mass penalty of about 50%. The second region, from about f=0.2 to 0.5 requires a very small additional mass penalty because the redesign is mostly achieved by the transfer of material from panel 2 to panel 1. Finally, for f>0.5 the undamaged design constraints have no influence on the damage-tolerant design and the thicknesses of the panels are completely determined by the damage-tolerance requirements. The damage-tolerant structure can carry a design load of 2fS when undamaged, which is larger than the required load S.

This simple two-bay example has very poor damagetolerance characteristics, mainly because the optimal design with no damage-tolerance constraints does not have any redundancy. In such cases, it may be more advantageous to change the design concept rather than provide the required damage tolerance by increasing the thicknesses of the panels. An alternate approach for achieving a better design is to add a middle horizontal stiffener to the left bay and remove the skin of the right bay as indicated in Fig. 3b. The skin material is now limited to the left bay and the damage condition is considered to be the destruction of either the top or the bottom half of that bay. The stiffener accomplishes two purposes: First, it localizes damage to one half of the remaining bay; second, it increases the buckling load. For simplicity, it is assumed that the mass of the stiffener is equal to the mass of the shear web material. Since each half of the bay carries a load of S/2 and is of length b/2 the thickness required for carrying the load S without buckling [see Eq. (9)] is

$$t' = [3Sb(1 - \nu^2) / \pi^2 \kappa' E]^{1/3}$$
 (13)

where κ' is the shear buckling coefficient corresponding to the new aspect ratio. The total mass including the stiffener is

$$m' = 2\rho abt' \tag{14}$$

If one part of the panel is damaged, the structure can still carry 50% of the load. For higher residual strength requirements (f>0.5), the thickness is

$$t' = [6fSb(1 - \nu^2) / \pi^2 \kappa' E]^{1/3}$$
 (15)

The dependence of the ratio of the mass of the damage-tolerant design to the mass of the reference design, m'/m_0 , on f is shown by the dashed curve in Fig. 5 for a/b = 1 ($\kappa = 9.34$, $\kappa' = 6.6$, Ref. 5). While the new configuration is heavier when no damage tolerance is required, it is the preferred design when significant residual strength is required.

For this simple example, a change in design concept may be required to prevent a large mass penalty for improved damage tolerance. When the structure is highly redundant, a change in design concept may not be necessary because there is a great

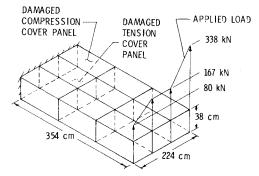


Fig. 5 Wing-box model.

degree of design flexibility afforded just by changing member sizes. In such a case, an automated optimization procedure may be used to obtain a good damage-tolerant design. This situation is illustrated by the next examples.

Wing-Box Design Examples

The two simple design examples discussed in the previous section demonstrate aspects of designing for damage tolerance and the associated mass penalty. Because structural redundancy is a major factor in damage tolerance, however, these examples are inadequate for representing more complex structures. The following wing-box examples are representative of a complex, highly redundant structure. Such structures are typically analyzed with the aid of finite-element computer programs.

The computer program used for obtaining minimum-mass designs for the wing-box examples is a modification of the WIDOWAC program.^{6,7} In the program, a wing-box structure is modeled by a finite-element representation that includes rod elements, constant-strain triangular membrane elements, and shear web elements. Quadrilateral elements made up of the average of four triangles are typically used for modeling cover panels. Design variables define thicknesses and areas of structural elements. The mathematical programing optimization procedure employs an efficient extended interior penalty function algorithm.

The modifications to the WIDOWAC program allow a model of the undamaged structural configuration and models of a number of damaged structural configurations to be analyzed simultaneously with independent loading conditions applied to each model. A damaged structural configuration is simulated by removing one or more finite elements from the undamaged structural model.

Description of Wing-Box Model

For this example, an unswept, untapered wing box with four spars and three ribs is modeled. The length, width, and depth dimensions of the wing box are shown in Fig. 5. The wing box is clamped at the root and subjected at the tip to the applied load distribution shown in Fig. 5. Upper and lower wing-skin panels are modeled by membrane finite elements, and webs of ribs and spars are modeled by shear web elements. Spar caps and vertical posts at the rib-spar intersections are modeled by rod elements. Two material systems are compared in this example: graphite-epoxy and an aluminum alloy. Material properties used in this study are given in Table 1. For the aluminum model, a single quadrilateral membrane element is used to represent a skin panel bounded by ribs and spars. For the graphite-epoxy model, four quadrilateral membrane elements, one for each of four material orientations, are stacked to form a laminate representing a skin panel bounded by ribs and spars. A balanced, symmetric laminate is generated internally in the computer program to represent the stacking sequence $(-45_i^{\circ}/+45_i^{\circ}/0_j^{\circ}/90_k^{\circ}/0_j^{\circ}/+45_i^{\circ}/-45_i^{\circ})$ for the composite panels. Here, i, j, and k are not integer numbers of plies, but are used to indicate that the thickness of each ply is controlled by a design variable. The +45- and -45-deg materials are controlled by a single design variable and the 0- and 90-deg materials by two independent design variables. Each rib and spar shear element in the graphite-epoxy model is represented by one quasi-isotropic graphite-epoxy shear web, and the spar caps are represented by 0-deg graphite-epoxy rod elements.

The finite-element models have 32 grid points and 72 degrees of freedom. The aluminum model has 75 finite elements and the graphite-epoxy model has 129. Design variables are distributed so that there is one design variable assigned to the thickness of each material orientation of each cover skin between each rib (i.e., constant chordwise distribution), one design variable assigned to the thickness of each spar web between ribs, one design variable assigned to

Table 1 Material properties and design allowables

	Aluminum	Graphite-epoxy
Longitudinal Young's modulus, GPa	72.3	130.9
Transverse Young's modulus, GPa	72.3	13.0
Shear modulus, GPa	27.6	6.4
Major Poisson's ratio	0.33	0.38
Density, kg/m ³	2770	1610
Stress allowables, MPa Normal Shear	503 290	···
Strain allowables Extensional Shear		0.004 0.015
Minimum gages Skin thickness, mm Web thickness, mm Cap areas, cm ²	0.51 0.51 0.65	0.28 ^a 0.56 0.65

^aThickness per lamina.

the area of each spar cap between ribs, and one design variable assigned to the thickness of each rib. The cross-sectional areas of the vertical posts at the rib-spar intersections are held constant for the models. A total of 45 design variables are used for the aluminum model, and 57 design variables are used for the graphite-epoxy model. The design constraints applied to the wing-box models are material-strength, minimum-gage, and panel-buckling constraints. The allowable values for the material-strength and minimum-gage constraints are given in Table 1. Each rib and spar shear web and each upper-surface cover-skin panel bounded by the ribs and spars are constrained not to buckle. The spar caps are also constrained so that their cross-sectional areas cannot exceed 3.9 cm².

Reference Designs and Their Residual Strength

Minimum-mass designs of the graphite-epoxy and aluminum wing boxes that were obtained without considering the effects of any potential damage conditions have total structural masses of 335 and 526 kg, respectively. These designs satisfy all material-strength, panel-buckling, and minimum-gage constraints imposed on the wing boxes for 100% of the applied load, as shown in Fig. 5.

The mass in kilograms and the percentage of the total mass of various components is given in Table 2 along with the total mass in kilograms for the reference designs. These results indicate that most of the structural mass of the designs is in the cover-skin panels and that the compression cover-skin panels need to be thicker than the tension cover-skin panels to satisfy the panel-buckling constraints imposed on the designs.

The sensitivity of the reference designs to potential damage conditions is represented by their residual strength after being damaged. In this study, the residual strength of a damaged structure is determined by removing the appropriate finite elements from the structural model to simulate the damage and then by determining the maximum percentage of the original applied load that can be carried by the damaged model without violating design constraints. The thickest cover-skin panels of the wing-box reference designs carry the highest internal loads, and damage to these panels is assumed to be a worst-case condition. To determine the effect of damage to these panels on the reference designs, two potential damage conditions are considered. For the first damage condition, the upper or compression cover-skin panel at the root between the first two spars (see Fig. 5) is removed. For

Table 2 Wass distribution in knograms and percentage of the total structural mass for wing-box componer	Table 2	Mass distribution in kilograms and	percentage of the total structural mass for wing-box component
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	Refer		Graphite-epo Compr panel da	ession	Tens panel da		Refer	ence,	Aluminun Compr panel da	ession	Ten panel da	sion amage,ª
Components	kg	9/0	kg	970	kg	070	kg	9/0	kg	970	kg	0/0
Compression cover-skin panels Tension	199	59	240	61	202	43	333	63	372	65	347	53
cover-skin panels	54 71	16 21	65 79	16 20	163 89	35 19	67 113	13 21	. 68 117	12 21	176 122	27 18
Shear webs Spar caps Total structure	7 . 335	2	9 395	20	9 466	2	9 526	2	10 571	2	9 657	1

^a Designed to carry 100% of the original applied load.

Table 3 Total mass in kilograms of minimum-mass designs that account for damage

Percent of original applied load	Compression pa	nel damage	Tension panel damage		
	Graphite-epoxy	Aluminum	Graphite-epoxy	Aluminum	
100	395	571	466	657	
90	375	558	441	622	
80	358	550	419	593	
70	346	541	397	569	
60	336	533	375	549	
50	335	527	359	532	
40	335	527	342	526	
30	335	527	335	526	

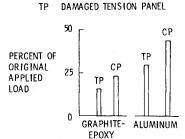
the second damage condition, the corresponding lower or tension cover-skin panel is removed.

The results of the residual strength analyses for the reference wing-box designs with a damaged compression or tension cover-skin panel are shown in Fig. 6 as the maximum percentage of the original applied load each damaged structure can carry without violating any design constraints. These results show that the graphite-epoxy reference design has lower residual strength than the aluminum reference design, and that damage to the tension cover-skin panel reduces the strength of the wing box more than damage to the compression cover-skin panel. In all four damaged cases, the rib shear web indicated in Fig. 7 buckles when the applied load exceeds the corresponding residual-strength result.

Minimum-Mass Damage-Tolerant Designs

The masses of the damage-tolerant designs for the graphiteepoxy and aluminum wing boxes considered in the referencedesign residual-strength study are given in Table 3. The additional mass (given as a percentage of the mass of the reference designs) needed to carry a given percentage of the original load applied to a damaged configuration is shown in Fig. 8. The open circles in Fig. 8 represent the results for the designs that account for the damaged compression cover-skin panel, and the open squares represent the results for the

Fig. 6 Residual-strength analysis results for reference wing-box designs with damaged tension and compression cover-skin panels.



DAMAGED COMPRESSION PANEL

designs that account for the damaged tension cover-skin panel. The residual-strength results for the reference designs with damaged cover panels are represented by the filled symbols on the abscissas of Fig. 8 for comparison. In each case, the residual strength of the design that accounts for potential damage is greater than the residual strength of the corresponding reference design. The differences in the open and filled symbols on the abscissas represent the increase in residual strength with no increase in structural mass when the influence of damage conditions is included in the design. For example, the residual strength of the graphite-epoxy wing box with compression cover-skin panel damage is increased from 23 to 60% of the original applied load when the effects of damage are included in the minimum-mass design process.

These remarkable increases in damage tolerance with no increase in mass are due to the nonuniqueness of the optimum designs. This phenomenon of several optima with approximately the same mass is especially pronounced for

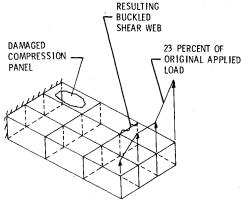


Fig. 7 Results of a residual-strength analysis of the reference undamaged graphite-epoxy design loaded to 23% of the original design load with a damaged compression cover-skin panel.

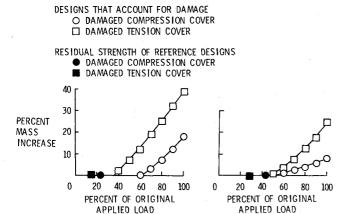


Fig. 8 Effect of designing for damage on residual strength and structural mass: a) graphite-epoxy; b) aluminum.

composite materials and has been observed previously;⁷ it is probably a result of the high degree of redundancy of the structure. Thus the designer has a powerful incentive to include damage-tolerance constraints from the outset so that the optimizer can select the minimum-mass design with the best damage-tolerance properties.

The mass in kilograms of the damage-tolerant designs and the percentage total mass of various components is given in Table 2 along with the total mass in kilograms for the heaviest damage-tolerant designs. Comparing the damage-tolerant results with the reference-design results indicates that the designs are more sensitive to the effects of damaged tension cover-skin panels than to the effects of damaged compression cover-skin panels.

The results of this example indicate that the damage tolerance of both the graphite-epoxy and aluminum wing boxes can be improved by including the influence of potential damage conditions in the design process. Comparing the aluminum design data with the composite design data in Fig. 8 indicates the following differences in design results.

- 1) When no damage-tolerance constraints are considered, the resulting composite design is more sensitive to damage than the aluminum design.
- 2) Greater improvements in residual strength with zero mass increment are possible for the composite design than for the aluminum design.
- 3) For very high levels of required damage tolerance, the composite design requires larger relative mass increments. (However, the graphite-epoxy designs are still much lighter than the corresponding aluminum designs.)

These three observations underscore the importance of designing a composite structure from the outset for the desired level of damage tolerance. An alternative approach is to design a structure without consideration of damage tolerance and then to redesign it to improve its damage tolerance properties. This approach is referred to as the sequential design approach in contrast to the simultaneous design approach used to obtain the results of Fig. 8. This approach is discussed in the next section.

Sequential Design Approach

A sequential design approach for providing a damage-tolerant design is to analyze a model of the potentially damaged structure and then to increase the thicknesses or cross-sectional areas of all overstressed or buckled components enough to provide the desired structural response. Damage-tolerant designs obtained by this sequential design approach for wing boxes required to support 100% of the original applied load with the damaged compression coverskin panel are used for comparison with the results obtained by the simultaneous design approach. The mass of the damage-tolerant aluminum wing box obtained by the sequential design approach is 577 kg (compared to 571 kg).

This aluminum design is essentially the same as the one obtained by designing for damage from the outset. The mass of the damage-tolerant graphite-epoxy design obtained by the sequential design approach is 452 kg, which is 14% heavier than the mass originally obtained by the simultaneous design approach. These results suggest that, for advanced-composite wing-box structures, designing for damage from the outset may lead to designs that are significantly lighter than designs obtained by the sequential design approach.

The two design approaches used for the wing-box example are parallel to the two approaches or design concepts for the simple two-bay deep-beam example. The sequential approach is the equivalent of trying to improve the damage tolerance of a fixed design concept. This approach was shown to result in a heavier design for the simple example. Including the damage constraints from the outset (i.e., a simultaneous design approach) permits the optimization procedure to change the design concept enough to accommodate the required damage tolerance with a smaller mass penalty.

Concluding Remarks

Two simple panel examples and more complex wing-box structure examples have been used to determine the mass penalty associated with imposing a given level of required damage tolerance on minimum-mass structural designs. The results obtained for these examples indicate that the mass penalty necessary to achieve a given level of damage tolerance is highly dependent on the redundancy of the structure. For statically determinate structures, a large mass penalty may be required to achieve even a small measure of damage tolerance by adding redundancy. However, for highly redundant structures, large improvements in damage tolerance may be obtained with little or no mass penalty by including damagetolerance constraints in the design process from the outset. If very severe damage-tolerance constraints are imposed on the design, they may become the only constraints that govern the design. In such cases, the mass penalty may be unacceptably high, and a change in structural design concept may be desirable.

The results from the wing-box examples suggest that applying the minimum-mass design process to structures made of advanced-composite materials appears to lead to designs that are more sensitive to damage than similarly designed minimum-mass structures made of metal. However, minimum-mass composite structures also appear to have a greater potential for improving damage tolerance with a small mass penalty than do metal structures. These observations suggest that it is more important to include a priori the influence of damage-tolerance constraints in the minimum-mass design of composite structures than it is for metal designs.

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